

**Effect of Medium Symmetries on Limiting the Number of Parameters
Estimated with Polarimetric SAR Interferometry**

Mahta Moghaddam

Jet Propulsion Laboratory, MS 300-227

4800 Oak Grove Drive, Pasadena, CA 91109, USA

Phone: (+1) (818) 354-1591; email: Mahta.Moghaddam@jpl.nasa.gov

Abstract

The addition of interferometric backscattering pairs to the conventional polarimetric synthetic aperture radar (SAR) data over forests and other vegetated areas increases the dimensionality of the data space, in principle enabling the estimation of a larger number of parameters. In this paper, the effect of **medium** symmetries on the number of independent interferometric measurements is studied. **Reflection**, **rotation**, and **azimuthal** symmetries are considered. In each case, the number of independent elements of the interferometric covariance matrix is derived, which puts an inherent limit on the number of scatterer parameters that can be estimated using single-baseline polarimetric interferometry.

1. Introduction

The addition of interferometric backscattering pairs to the conventional polarimetric SAR data over forests and other vegetated areas increases the dimensionality of the data space, in principle enabling the estimation of a larger number of vegetation parameters. A conventional polarimetric SAR system transmits and receives vertical (v) and horizontal (h) polarizations at a single antenna [1]. Images are formed by correlating various available combinations of colocated transmit-receive pairs. In polarimetric interferometry, however, v - or h -polarized signals are transmitted at one antenna but received at two antennas separated by a baseline B (Figure 1). Here, we consider single-pass interferometry only. The interferometric images are formed by cross-correlating two transmit-receive pairs, each received at a different end of the baseline [2-3]. Hence, conventional polarimetry is a special case of the interferometric configuration, where the baseline is zero. The issue of sensitivity of these data to vegetation scattering parameters aside [4-8], this paper poses the question: Will increasing the number of data channels as such result in a one-to-one increase in the number of parameters that can be estimated, or do vegetation and data properties inherently limit that number otherwise? The answer to this question is derived on a strictly theoretical basis, and the implications for practical applications are discussed.

A complete set of polarimetric SAR correlation data at a given frequency and a given incidence angle can be described with the covariance, or coherency, matrix $\overline{\mathbf{C}}$, formed by correlating the scattering matrix elements and normalizing the results [1,4,9,10]:

$$\overline{\mathbf{C}} = \lim_{A \rightarrow \infty} \frac{4\pi}{A} \langle \mathbf{S} \mathbf{S}^\dagger \rangle, \quad \mathbf{S} = (s_{hh} \quad s_{hv} \quad s_{vh} \quad s_{vv})^t, \quad (1)$$

where A is the illuminated area and \dagger indicates conjugate transpose. The matrix \mathbf{S} is the scattering matrix. In the polarimetric interferometric SAR (POL/INSAR) case, assuming transmitting at only one end of the baseline and receiving at both ends, the covariance matrix can be similarly obtained by cross-correlating the elements

of the scattering matrix for each end of the baseline. This can be written as

$$\overline{\mathbf{C}} = \begin{pmatrix} \sigma_{hh_1hh_2} & \sigma_{hh_1hv_2} & \sigma_{hh_1vv_2} \\ \sigma_{hv_1hh_2} & \sigma_{hv_1hv_2} & \sigma_{hv_1vv_2} \\ \sigma_{vv_1hh_2} & \sigma_{vv_1hv_2} & \sigma_{vv_1vv_2} \end{pmatrix}, \quad (2)$$

where σ denotes the normalized backscattering cross sections for the correlation pairs at all possible polarization combinations, and subscripts 1 and 2 refer to the two ends of the interferometric baseline. Note that it is assumed that the media under study are reciprocal and therefore the reciprocity principle has already been used in $\overline{\mathbf{C}}$ to eliminate the correlation pairs containing vh at either end of the baseline. In general, the nine complex elements of this matrix are independent, resulting in 18 independent data points. The zero-baseline polarimetric covariance matrix is Hermitian and provides an additional 9 mathematically independent measurements, resulting in a total of 27 data points in the general case.

In this paper, various symmetry properties of the scattering medium are used to study whether any of the above nine interferometric cross-correlations can be further eliminated. The number of independent pairs has direct consequences in their utility in parameter estimation schemes, since the maximum number of parameters that can be estimated cannot exceed the number of unique measurements. In the following, the independent components of the POL/INSAR data are derived for media with reflection, rotation, and azimuth symmetries. Similar derivations have been carried out previously for simple polarimetry, i.e., zero baseline [9,11]. This paper extends those to the interferometric case of general nonzero baselines.

2. Derivation of Independent POL/INSAR Pairs

In this section, the effect of reflection, rotation, and azimuth symmetries of the medium on eliminating redundant interferometric backscattering coefficients in $\overline{\mathbf{C}}$ is investigated. The medium symmetries are defined with respect to the orthogonal right-handed coordinate system given by the unit vectors \hat{h} , \hat{v} , and \hat{k} , the latter being the direction of wave incidence (propagation).

2.1. Reflection symmetry

Consider a medium with reflection symmetry about a plane containing the direction of incidence \hat{k} , and one of the polarization directions, e.g., \hat{v} , as shown in Figure 2. An example of such a medium in SAR applications is an agricultural field plowed in the direction parallel to the flight line. In Figure 2, assume that the incident wave is rotated by any angle α about the plane of symmetry, resulting in new polarization vectors \hat{h}' and \hat{v}' . Further assume that the wave is transmitted with \hat{h}' -polarization and received at either \hat{h}' or \hat{v}' polarizations, so that the correlation pairs $\sigma_{hh_1hv_2}(\alpha)$ and $\sigma_{hv_1hh_2}(\alpha)$ are available. Now consider the mirror images of \hat{h}' and \hat{v}' about the axis \hat{h} , and call them \hat{h}'' and \hat{v}'' . Due to the reflection symmetry of the medium, if now the wave is transmitted with \hat{h}'' polarization and received at either \hat{h}'' or \hat{v}'' polarizations, the data should be identical to the $\hat{h}' - \hat{v}'$ measurements above. However, note that in order to have consistent definitions and a right-handed coordinate system, the horizontal and vertical polarizations now have to be interchanged. This results in polarization vectors \hat{h}''' (previously called \hat{v}'') and \hat{v}''' (previously called \hat{h}''). Hence, we get

$$\sigma_{v'''v'''_1v'''h'''_2} = \sigma_{h'h'_1h'v'_2}. \quad (3a)$$

$$\sigma_{v'''h'''_1v'''v'''_2} = \sigma_{h'v'_1h'h'_2}. \quad (3b)$$

From Figure 2, the vectors \hat{h}''' and \hat{v}''' are simply equivalent to the original \hat{h} and \hat{v} vectors, rotated by $\pi/2 - \alpha$. Hence, the following conditions are satisfied:

$$\sigma_{vv_1vh_2}(\pi/2 - \alpha) = \sigma_{hh_1hv_2}(\alpha), \quad (4a)$$

$$\sigma_{vh_1vv_2}(\pi/2 - \alpha) = \sigma_{hv_1hh_2}(\alpha). \quad (4b)$$

To obtain the elements of $\overline{\mathbf{C}}$ after rotation by the angle α , first the rotated scattering matrix is derived by applying the operator $\overline{\mathbf{R}}$ for any angle α

$$\overline{\mathbf{R}} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}, \quad (5)$$

$$\overline{\mathbf{S}}(\alpha) = \overline{\mathbf{R}}^{-1} \overline{\mathbf{S}}|_{\alpha=0} \overline{\mathbf{R}}, \quad (6)$$

where

$$\overline{\mathbf{S}}|_{\alpha=0} = \begin{pmatrix} s_{hh} & s_{hv} \\ s_{vh} & s_{vv} \end{pmatrix}. \quad (7)$$

Applying the rotations in Equations (6) and (7) to the corresponding elements of the POL/INSAR covariance matrix in Equation (1) and completing the algebra as shown in Appendix A, we obtain

$$\overline{\mathbf{C}}_{refl} = \begin{pmatrix} \sigma_{hh_1hh_2} & 0 & \sigma_{hh_1vv_2} \\ 0 & \sigma_{hv_1hv_2} & 0 \\ \sigma_{vv_1hh_2} & 0 & \sigma_{vv_1vv_2} \end{pmatrix}. \quad (8)$$

The resulting covariance matrix contains five independent complex elements, or 10 data points. If the baseline is zero, i.e., the two ends of the baseline coincide as is the case for conventional polarimetry, the diagonal terms are purely real, and σ_{hhvv} and σ_{vvhh} are complex conjugates. This yields five unique data points. Since the zero-baseline configuration is, by default, always available in an interferometric system, the total number of unique data points is 15. This is contrasted to the general case, where there are 27 unique data points.

2.2. Rotation symmetry

If a medium is invariant under any rotation about an axis parallel to the direction of wave propagation as shown in Figure 3, it is said to have rotation symmetry about that axis. In this case, the following condition has to be satisfied for any angle α :

$$\sigma_{mn_1pq_2}(\alpha) = \sigma_{mn_1pq_2}(0), \quad (9)$$

where m, n, p, q are h or v polarizations. A uniformly random medium comprised of discrete helical scatterers is a general example of a rotationally symmetric medium. Applying the rotation operator given by Equation (5) to elements of $\overline{\mathbf{C}}$ in Equation (1) and equating them to nonrotated elements results in (see Appendix B)

$$\overline{\mathbf{C}}_{rot} = \begin{pmatrix} \sigma_{hh_1hh_2} & \sigma_{hh_1hv_2} & \sigma_{hh_1hh_2} - 2\sigma_{hv_1hv_2} \\ -\sigma_{hh_1hv_2} & \sigma_{hv_1hv_2} & \sigma_{hh_1hv_2} \\ \sigma_{hh_1hh_2} - 2\sigma_{hv_1hv_2} & -\sigma_{hh_1hv_2} & \sigma_{hh_1hh_2} \end{pmatrix}. \quad (10)$$

In this case, the covariance matrix $\overline{\mathbf{C}}_{rot}$ contains only three independent complex elements, or six data points. If the baseline is zero, σ_{hhhh} and σ_{hvvv} are purely real, and it can be shown that σ_{hhhv} is purely imaginary [11], resulting in three additional data points. Hence the total number of available POL/INSAR observations is 9.

2.3. Azimuthal symmetry

When both reflection and rotation symmetries are present, the medium is said to have azimuthal symmetry and satisfies the conditions of both reflection and rotation symmetries. For example, a uniformly random medium with spherical scatterers or a random rough surface at normal incidence have azimuthal symmetry. The resulting POL/INSAR covariance matrix, therefore, is obtained from combining Equations (8) and (10):

$$\overline{\mathbf{C}}_{az} = \begin{pmatrix} \sigma_{hh_1hh_2} & 0 & \sigma_{hh_1hh_2} - 2\sigma_{hv_1hv_2} \\ 0 & \sigma_{hv_1hv_2} & 0 \\ \sigma_{hh_1hh_2} - 2\sigma_{hv_1hv_2} & 0 & \sigma_{hh_1hh_2} \end{pmatrix}. \quad (11)$$

Here, there are only two independent elements in the covariance matrix, or four unique polarimetric, interferometric data points. For conventional polarimetry, i.e.,

zero baseline, σ_{hhhh} and σ_{hvhv} are both real, so that there are only two additional unique measurements. The total POL/INSAR data points is six.

3. Discussion and Summary

Symmetries of a given medium can put a limit on the number of independent measurements that can be obtained using polarimetric interferometry. Natural scatterers, such as forests, oceans, and bare soil surfaces often display one or more of the symmetry properties discussed above, and hence the limitations discussed here apply to POL/INSAR data over these areas. Depending on the type of symmetries present, the number of independent available measurements that can be used to estimate medium parameters will vary. It was shown that, whereas in the general case there are 27 mathematically independent measurements possible from a polarimetric interferometer, this number can be reduced to 15, 9, and 6 if the medium has reflection, rotation, or azimuthal symmetries, respectively. The results can be used in several ways in the interpretation of SAR data and the development of parameter estimation schemes:

1. If fully polarimetric interferometric data are available, symmetry properties of the medium could be deduced from the data. In other words, the POL/INSAR covariance matrix can be used as a classification tool. Although a similar statement can be made about zero-baseline polarimetric data, the introduction of interferometry to the conventional polarimetry increases the dimensionality of the data space which is used to determine the symmetry properties, and hence, is expected to significantly increase the accuracy of such assessment. In the cases studied in this paper, POL/INSAR data provide twice the number of data points that would be available from conventional polarimetry. The increased classification accuracy possible using POL/INSAR data is currently under investigation using the recently acquired data by the NASA/JPL AIRSAR in the POLTOP mode, and will be reported in future works.
2. If symmetry properties of the medium are known, e.g., if it is known that a

certain scene contains primarily dense forests or bare soil surfaces, the required complexity of the observation scheme can be reduced by performing only the necessary measurements. For example, if a scene is occupied by a dense forest canopy whose scatterers have no orientation preference, there is no need to transmit both the h- and v-polarized signals. Transmitting at one polarization and receiving at both will result in a complete representation of the covariance matrix, assuming sufficient calibration. On the other hand, if both transmit and both receive polarization channels are available, they can be used, along with the known symmetry properties of the medium, as a calibration tool for the polarimetric, interferometric data set [12,13]. Similar to the statements in item 1 above, using POL/INSAR data as opposed to only the zero-baseline polarimetric SAR data should, in principle, result in significant enhancement of such a calibration procedure.

3. The number of unique data points should be used as an upper bound for the number of parameters used in formulating the scattering models if those models are to be used for estimating medium parameters from polarimetric interferometric data. The actual number of model parameters will be, in general, even lower since although the measurements could be mathematically independent, they may not necessarily be physically sensitive to medium parameters. The results derived in this paper are not limited to any specific physical model that might be used to describe the medium. However, such models can be used to investigate further limitations in parameter estimation due to lack of sensitivity of various POL/INSAR channels to the unknown parameters.

Symmetries of a medium do enable a more simplified description, hence reducing the number of parameters needed in scattering models of the medium. However, almost always the parameters still outnumber the measurements, and further assumptions and simplifications may be needed to limit the model parameter set to avoid an underdetermined estimation problem.

APPENDIX A

The elements of the rotated scattering matrix $\bar{S}(\alpha)$ can be obtained from Equation (6) after substituting Equations (5) and (7). This gives

$$S_{hh}(\alpha) = S_{hh}(0) \cos^2 \alpha + 2S_{hv}(0) \sin \alpha \cos \alpha + S_{vv}(0) \sin^2 \alpha, \quad (A1)$$

$$S_{hv}(\alpha) = -S_{hh}(0) \sin \alpha \cos \alpha + S_{hv}(0)(\cos^2 \alpha - \sin^2 \alpha) + S_{vv}(0) \sin \alpha \cos \alpha, \quad (A2)$$

$$S_{vv}(\alpha) = S_{hh}(0) \sin^2 \alpha - 2S_{hv}(0) \sin \alpha \cos \alpha + S_{vv}(0) \cos^2 \alpha. \quad (A3)$$

In the presence of a medium with reflection symmetry about the direction of wave propagation, the backscattering coefficients must satisfy the following conditions:

$$\sigma_{vv_1vh_2}(\pi/2 - \alpha) = \sigma_{hh_1hv_2}(\alpha), \quad (A4)$$

$$\sigma_{vh_1vv_2}(\pi/2 - \alpha) = \sigma_{hv_1hh_2}(\alpha). \quad (A5)$$

From Equations (2) and (A1)-(A3), we can write for Equation (A4)

$$\begin{aligned} \sigma_{vv_1vh_2}(\pi/2 - \alpha) = & \cos^4 \alpha (-\sigma_{hh_1hv_2}) + \\ & \cos^3 \alpha \sin \alpha (-\sigma_{hh_1hh_2} + \sigma_{hh_1vv_2} + 2\sigma_{hv_1hv_2}) + \\ & \cos^2 \alpha \sin^2 \alpha (\sigma_{hh_1hv_2} + 2\sigma_{hv_1hh_2} - 2\sigma_{hv_1vv_2} - \sigma_{vv_1hv_2}) + \\ & \sin^3 \alpha \cos \alpha (\sigma_{vv_1vv_2} - \sigma_{vv_1hh_2} - 2\sigma_{hv_1hv_2}) + \\ & \sin^4 \alpha \sigma_{vv_1hv_2}, \end{aligned} \quad (A6)$$

and

$$\begin{aligned} \sigma_{hh_1hv_2}(\alpha) = & \cos^4 \alpha (\sigma_{hh_1hv_2}) + \\ & \cos^3 \alpha \sin \alpha (-\sigma_{hh_1hh_2} + \sigma_{hh_1vv_2} + 2\sigma_{hv_1hv_2}) + \\ & \cos^2 \alpha \sin^2 \alpha (-\sigma_{hh_1hv_2} - 2\sigma_{hv_1hh_2} + 2\sigma_{hv_1vv_2} + \sigma_{vv_1hv_2}) + \\ & \sin^3 \alpha \cos \alpha (\sigma_{vv_1vv_2} - \sigma_{vv_1hh_2} - 2\sigma_{hv_1hv_2}) + \\ & \sin^4 \alpha (-\sigma_{vv_1hv_2}). \end{aligned} \quad (A7)$$

Since Equations (A4) and (A5) must hold for all angles α , the coefficients multiplying each trigonometric function of α in Equations (A6) and (A7) must be equal. In particular,

$$-\sigma_{hh_1hv_2} = \sigma_{hh_1hv_2} \quad \text{or} \quad \sigma_{hh_1hv_2} = 0, \quad (A8)$$

$$\sigma_{vv_1 hv_2} = -\sigma_{vv_1 hv_2} \quad \text{or} \quad \sigma_{vv_1 hv_2} = 0. \quad (A9)$$

Similarly, Equation (5) results in the following conclusion:

$$\sigma_{hv_1 hh_2} = 0, \quad (A10)$$

$$\sigma_{hv_1 vv_2} = 0. \quad (A11)$$

Substituting these results in Equation (1), we obtain Equation (8) in Section 2.1.

APPENDIX B

For a rotationally symmetric medium, Equation (9) must hold for all polarizations of the backscattering coefficient. In particular,

$$\sigma_{hh_1 hh_2}(0) = \sigma_{hh_1 hh_2}(\alpha), \quad (B1)$$

$$\sigma_{hh_1 hv_2}(0) = \sigma_{hh_1 hv_2}(\alpha), \quad (B2)$$

Using Equations (A1) and (A2) of Appendix A, we get from Equation (B1)

$$\begin{aligned} \sigma_{hh_1 hh_2} = \sigma_{hh_1 hh_2}(\alpha) = & \cos^4 \alpha (\sigma_{hh_1 hh_2}) + \\ & \cos^3 \alpha \sin \alpha (2\sigma_{hh_1 hv_2} + 2\sigma_{hv_1 hh_2}) + \\ & \cos^2 \alpha \sin^2 \alpha (\sigma_{hh_1 vv_2} + 4\sigma_{hv_1 hv_2} + \sigma_{vv_1 hh_2}) + \\ & \sin^3 \alpha \cos \alpha (2\sigma_{hv_1 vv_2} + 2\sigma_{vv_1 hv_2}) + \\ & \sin^4 \alpha (\sigma_{vv_1 vv_2}). \end{aligned} \quad (B3)$$

The above equation must hold for all angles α , and hence, the factors multiplying each trigonometric function of α on either side of the equation must be equal. The results are

$$\sigma_{hh_1 hh_2} = \sigma_{vv_1 vv_2}, \quad (B4)$$

$$\sigma_{hh_1 hv_2} = -\sigma_{hv_1 hh_2}, \quad (B5)$$

$$\sigma_{vv_1 hv_2} = -\sigma_{hv_1 vv_2}. \quad (B6)$$

Following a similar procedure, Equation (B2) gives

$$\sigma_{hh_1 hv_2} = \sigma_{hv_1 vv_2}, \quad (B7)$$

$$\sigma_{hh_1hv_2} = -\sigma_{vv_1hv_2}, \quad (B8)$$

$$\sigma_{vv_1hv_2} = -\sigma_{hv_1vv_2}. \quad (B9)$$

Note that Equations (B6) and (B9) are identical. Performing the same steps for other polarizations will produce the same results again, and therefore, need not be repeated. Substituting the above results into Equation (1), we obtain Equation (10) in Section 2.2.

References

1. Zebker, H., and J. van Zyl, "Imaging radar polarimetry: A review," *Proc. IEEE*, vol. 79, no. 11, pp. 1583-1606, 1991.
2. Rodriguez, E., and J. Martin, "Theory and design of interferometric synthetic aperture radars," *IEE Proc. F*, 139, pp. 147-159, 1992.
3. Zebker, H. et al., "The TOPSAR interferometric radar topographic mapping instrument," *IEEE Trans. Geosci. Remote Sensing*, vol. 30, pp. 933-940, 1992.
4. Cloude, S. R., and K. P. Papathanassiou, "Polarimetric SAR Interferometry", *IEEE Trans. Geosci. Remote Sensing*, in press, 1998.
5. Treuhaft, R., S. Madsen, M. Moghaddam, and J. van Zyl, "Vegetation characteristics and underlying topography from interferometric radar," *Rad. Sci.*, vol. 31, no. 6, pp. 1449-1485, 1996.
6. Floury, N., D. Dendal, T. Le Toan, and J. Souyris, "Modeling of radar response of some land cover types for the interpretation of polarimetric/interferometric measurements," *PIERS'98*, Nantes, France, 1998.
7. Hoekman, D., and C. Verekamp, "High-resolution polarimetric and interferometric radar observation of tropical rain forest," *PIERS'98*, Nantes, France, 1998.

8. Treuhaft, R., and M. Moghaddam, "A unified analysis of radar interferometry and polarimetry for the estimation of forest parameters," *Proc. IGARSS'98*, Seattle, WA, 1998; also presented at PIERS'98, Nantes, France, 1998.
9. H.C. van de Hulst, *Light Scattering by Small Particles*,. New York: Dover Publications, Inc., 1957.
10. Tsang, L., Kong, J., and R. Shin, *Theory of Microwave Remote Sensing*. New York: Wiley, 1985.
11. S. V. Nghiem, S. H. Yueh, R. Kwok, and F. K. Li, "Symmetry properties in polarimetric remote sensing," *Rad. Sci.*, vol. 27, no. 5, pp. 693-710, 1992.
12. Klein, D. J., and A. Freeman, "Quadpolarization SAR calibration using target reciprocity," *J. Electromagnetic Waves Appl.*, vol. 5, no. 7, pp. 735-751, 1991.
13. Quegan, S., "A unified algorithm for phase and cross-talk calibration of polarimetric data - Theory and observations," *IEEE Trans. Geosci. Remote Sensing*, vol. 32, no. 1, pp. 89-99, 1994.

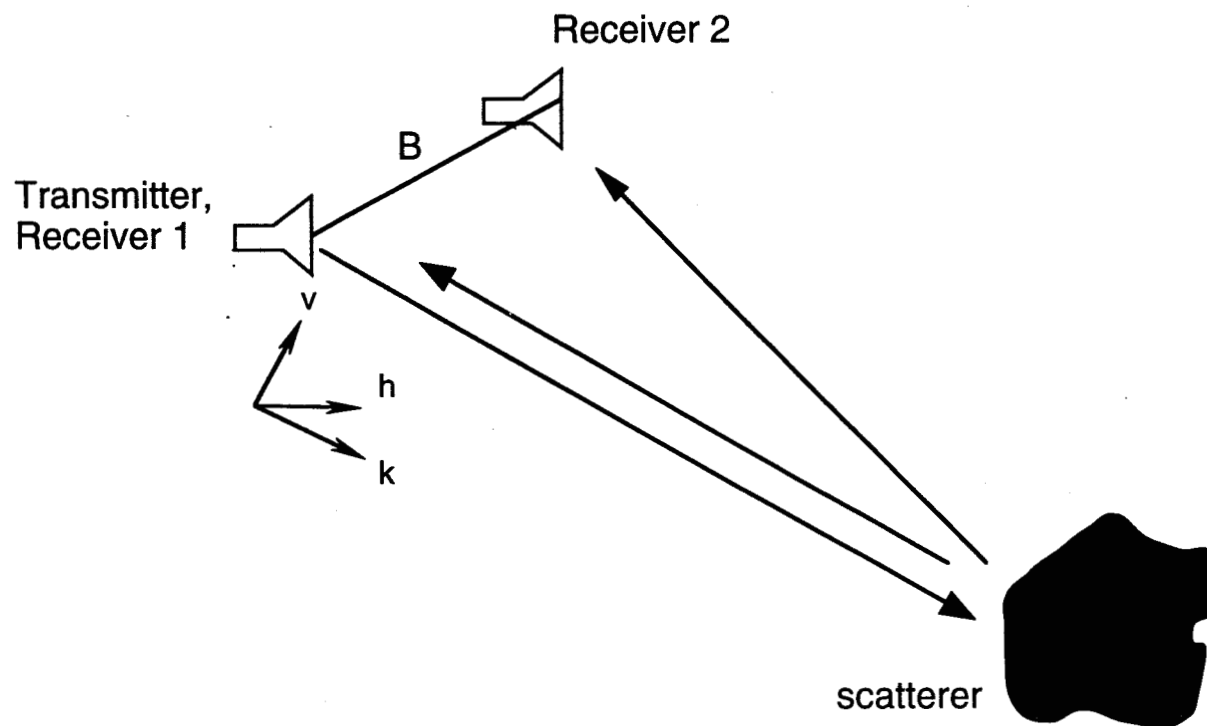


Figure 1. Single-pass interferometric data acquisition geometry.

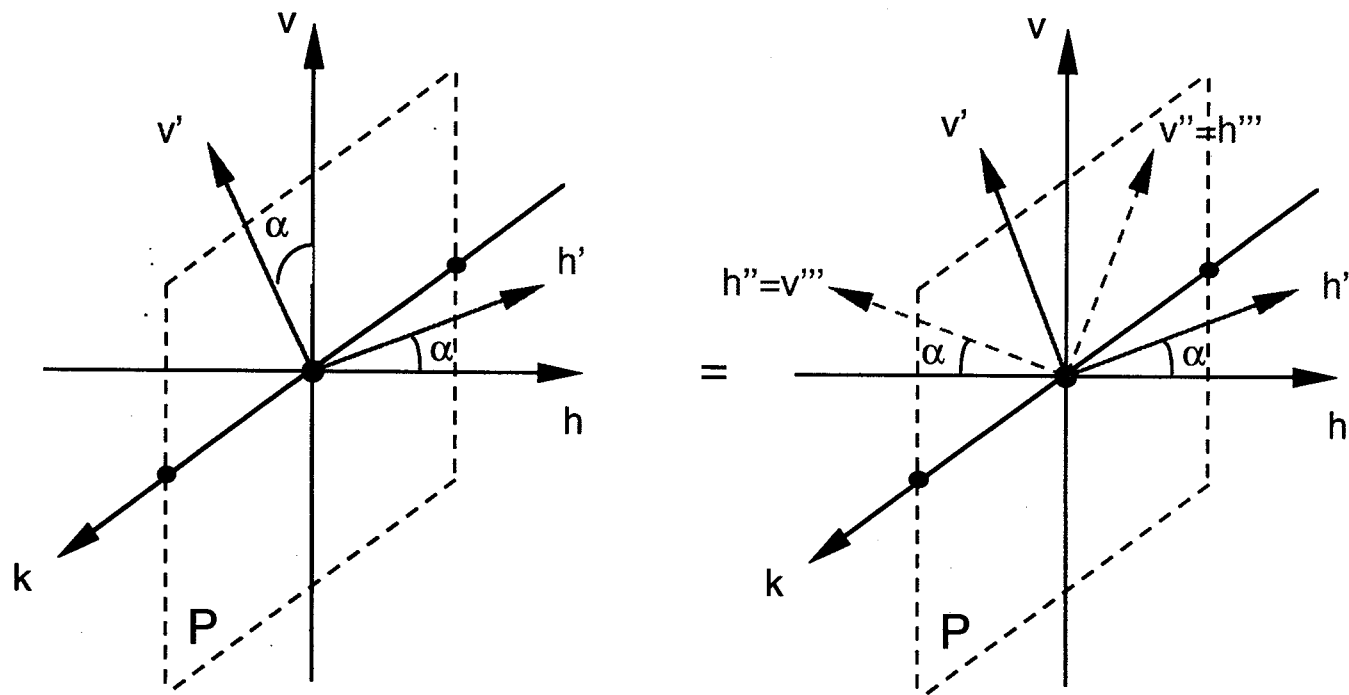


Figure 2. Reflection symmetry about the Plane P results in equivalent measurements after a reflection of the polarization vectors about the plane of symmetry.

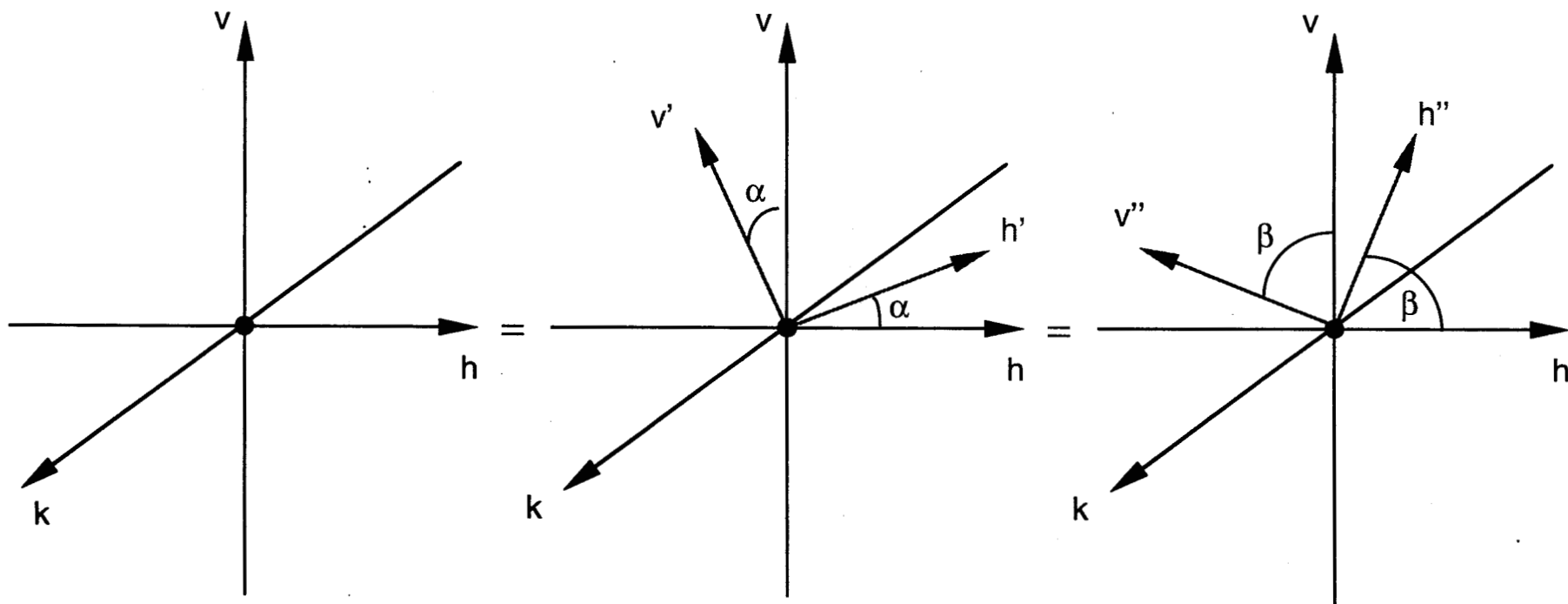


Figure 3. Rotation symmetry about the direction of propagation leaves the polarimetric measurements unchanged after a rotation of the polarization vectors by any angle.